# Measurement of Blended Objects in LSST

# Jim Bosch for LSST Data Management

Thursday 25<sup>th</sup> August, 2016

### 1 Introduction

Most LSST objects will overlap one or more of its neighbors enough to affect naive measurements of their properties. One of the major challenges in the deep processing pipeline will be measuring these sources in a way that corrects for and/or characterizes the effect of these blends.

The measurements of interest are those in §5.2 of the DPDD. These can be split up broadly into two categories:

- weighted moments (includes adaptive moments, Kron and Petrosian fluxes, and aperture surface brightness;
- forward modeling (includes point source model fit and bulge-disk model fit).

Most measurements that involve the PSF or a PSF-convolved function as a weight function can be interpreted in either way (this includes algorithms for *standard colors* and probably *centroids*), but must be treated as weighted moments in calculating their uncertainties (i.e. per-pixel variances must not be used in the weighting) in order to produce unbiased fluxes.

The statistical framework in which weighted moments make sense assumes that each object is isolated from its neighbors. As a result, our only option for these measurements is *deblending*, which we define here as any procedure that attempts to remove neighbors from the pixel values prior to measurement.

In forward modeling, we convolve a model for the object with our model for the PSF, compare this model to the data, and either optimize to find best-fit parameters or explore the full likelihood surface in another way (e.g. Monte Carlo sampling). We can use the deblending approach for forward fitting, simply by fitting each object separately to the deblended pixels. However, we can also use *simultaneous fitting*, in which we optimize or sample the models for multiple objects jointly.

Both deblending and simultaneous fitting have some advantages and disadvantages:

- Deblending provides no direct way to characterize the uncertainties in an object's measurements due to neighbors, while these are naturally captured in the full likelihood distribution of a simultaneous fit. This likelihood distribution may be very high-dimensional in a fit that involves many objects, however, and may be difficult to characterize or store.
- Deblending generally allows for more flexible morphologies than the analytic models typically used in forward fitting, which is particularly important for nearby galaxies and objects blended with them; simultaneous fitting is only statistically well-motivated to the extent the models used can reproduce the data.
- Once deblended pixels are available, fitting objects simultaneously will almost always be more computationally expensive than fitting them separately to the deblended pixels. At best, simultaneous fitting will have similar performance but still require more complex code. And because we will need to deblend pixels to support some measurement algorithms, we'll always have to deblend whether we want to subsequently do simultaneous fitting or not.

This paper focuses on the problem of blended measurement only; the details of how we deblend pixels and the models and algorithms we use in simultaneous fitting will be described elsewhere.

# 2 Blend Families and Footprints

We identify groups of blended objects at the detection stage from their isophotes at the detection limit; a single simply-connected region of above-threshold pixels is a *parent*, with *children* initially discovered as peaks within that region. These peaks may not originate in the same image; we will merge peaks from multiple detection images (e.g. coadds of different filters or ranges of observation dates).

We call the above-threshold region (and the data structure that defines it) a Footprint, and the combination of such a region with the values of the pixels within it (either original data or deblended) a HeavyFootprint.

For each blend family, in addition to measuring the properties of the children (either via deblending or simultaneous fitting), we also measure the parent: we interpret the region as a single unblended object. This is essentially an incomplete but useful hedge against overdeblending (we'd like to evaluate all alternate hypotheses for any combination of peaks belonging to the same object, but that's infeasible).

## 3 Deblended Measurement

The specific approach to deblending we plan to take for LSST is based on the deblender in the SDSS *Photo* pipeline.

Given pixel values  $I_i$ , we create a "template"  $T_{i,j}$  that represents an attempt to model the surface brightness of object j at pixel i.<sup>1</sup> We then find the best least-squares linear combination of templates to the data (ignoring per-pixel variances), solving for coefficients  $\alpha_j$ :

$$\alpha = (T^T T)^{-1} T^T I \tag{1}$$

Our best-fit prediction for the pixel value vector is thus  $T\alpha$ . Because this is not the same as the true pixel vector I, we do not use the per-object model prediction  $T\alpha$  directly for the deblended pixel values; instead we reapportion the true per-pixel fluxes according to the relative predicted contribution from each object:

$$D_{i,j} = \frac{T_{i,j} \alpha_j}{\sum_{k} T_{i,k} \alpha_k} I_i \tag{2}$$

While these deblended pixel values  $D_{i,j}$  are what we store in the HeavyFootprint for each child object, we do not perform measurements on these values directly for two reasons:

- $D_{i,j}$  typically has many zero entries, especially for a large blend family (i.e. many pixels for which a particular object has no contribution). We include only the nonzero values in the HeavyFootprints for efficiency reasons.
- Many measurements utilize pixels beyond the blend family's Footprint, and in fact may extend to pixels that are in another family.

To address these issues, we measure deblended objects using the following procedure:

- 1. Replace every above threshold pixel in the image (all Footprints) with randomly generated noise that matches the background noise in the image.
- 2. For each blend family:
  - (a) For each child object in the current blend family:
    - i. Insert the child's HeavyFootprint into the image, replacing (not adding to) any pixels it covers.
    - ii. Run all measurement algorithms to produce *child* measurements.
    - iii. Replace the pixels in the child's Footprint region with (the same) random noise again.
  - (b) Revert the pixels in the parent Footprint to their original values.
  - (c) Run all measurement algorithms to produce parent measurements.

<sup>&</sup>lt;sup>1</sup> In *Photo*, this template was determined from symmetry arguments and a number of heuristics; a full description of how we plan to generate templates in LSST is beyond the scope of this paper.

(d) Replace the parent Footprint pixels with (the same) random noise again.

This procedure double-counts flux that is not part of a Footprint, but this is considered better than ignoring this flux, because most measurement algorithms utilize some other procedure for downweighting the contribution of faraway pixels.

## 4 Simultaneous Fitting

### 4.1 Model Selection

The LSST pipeline will fit both a moving point source and a galaxy model to each object (see §5.2.1 of the DPDD). With simultaneous fitting, however, we also have to consider which models to use for neighbors, and it is clear that allowing for every possible combination is infeasible (we'd need to fit each blend  $2^N$  times, where N is the number of objects in the blend).

One possible alternative would be to determine the best model for each object from separate fitting done on deblended pixels, and then fit simultaneously using just those models. This makes the simultaneous fitting essentially useless for classification purposes, however, and it doesn't reflect the reality that many objects will be impossible to securely classify.

Another option would be to do a single simultaneous fit using a hybrid model that transitions between a moving point source and a galaxy model. Because a non-moving point source is a limit shared by both models, the transition is continuous, and it should be possible to fit using both sampling methods and optimization algorithms with some modification (though it will probably be impossible to use general-purpose algorithms without modification, given the complexity of the parameter constraints).

### 4.2 Minimization

Simultaneous fitting using optimization algorithms is straightforward from a mathematical standpoint, but potentially difficult from a computational and storage standpoint.

Nearly all numerical optimization algorithms involve a matrix factorization for which the computation complexity is  $O(N^2)$  in the number of parameters, and this makes the worst-case performance scale with the square of the number of objects in the blend (since the number of total parameters scales linearly with the number of objects being fit together). This matrix is typically sparse for extremely large blends, so sparse matrix methods may avoid this problem (at an additional cost in overhead). It is also worth noting that while the limiting performance for extremely large blends may go as  $O(N^2)$ , the bottleneck in fitting galaxies is generally the evaluation of the models and their first derivatives, which is just O(N) in the number of parameters.

Optimization-based fitting typically includes an estimate of the parameter covariance matrix as one of its outputs, and in simultaneous fitting this covariance matrix naturally includes cross-object terms. These terms are, of course, how we characterize how our uncertainty of an object's properties is affected by its neighbors, and hence are in some sense the reason we're doing simultaneous fitting at all. These terms don't fit naturally within the usual catalog/database model, in which one row corresponds to a single object. The cross-object terms would need to be stored in some other way, making them more difficult for users to access. Perhaps more importantly, for large blends the total number of outputs is  $O(N^2)$  in the number of objects. The matrix should be sparse for sufficiently large blends, however, so a storage scheme that takes advantage of this would address the problem.

If we elect to use hybrid models described in 4.1, we will almost certainly have to develop our own optimization code rather than adopt an existing third-party code. High-quality optimization libraries that can handle complex parameter constraints are extremely rare, and generally focused on a very specific domain. It is likely we'd have to develop our own optimizer even for single-object, non-simultaneous fitting, however, as even the simpler constraints involved in a single-object galaxy models are sufficiently complex to give most free optimizers trouble.

## 4.3 Monte Carlo Sampling

One of the advantages of Monte Carlo methods is that they scale better than optimization methods as dimensionality increases. If we consider the samples themselves to be the output of such an algorithm, the storage and catalog problems we encountered for optimizer outputs simply don't occur: if we sample simultaneously from a multi-object posterior, we can simply split the storage and representation of those samples across objects: they'll be the same samples, but each object's storage will only include its own parameters. Taken together, the samples represent the joint posterior for all objects in a blend; taken separately, they represent the marginal posteriors.

The scaling with dimensionality for most Markov Chain Monte Carlo algorithms depends strongly on the nature of the distribution itself. The burn-in period for such algorithms is typically thousands of samples, however, which makes them impractical for our problem, for which we can only evaluate approximately 200 samples per object (at least when fitting to multi-epoch data). Instead, our baseline plan is to use adaptive importance sampling, in which we draw samples from an analytic distribution that we construct to approximate the true distribution, then weight those samples according to the true distribution. We can then use those weighted samples to modify the analytic distribution in an iterative sense. Most importantly, we can do most of these iterations using a fast approximation to the true likelihood (by fitting to a coadd instead of multiple epochs, or using a fast but inexact convolution).

Unfortunately, this means we need to construct, draw from, and update a high-dimensional analytic distribution (typically a mixture Gaussians or Student's T), and these operations are typically  $O(N^2)$  in the total number of parameters. As with optimizers, these operations are nearly always subdominant to the time required to evaluate the likelihood itself. Instead, it is the potential complexity of the likelihood surface for large blends that is most concerning, especially when hybrid models are considered. In principle, anything can be done with mixture distributions, but it remains an open question how efficient this approach will be.

## 5 Divide and Conquer for Large Blends

Regardless of the efficiency of our algorithms in the large-blend limit, it will be necessary to split the very largest blends and handle them iteratively. This will happen automatically, of course, at the boundaries of our sky pixellization scheme, as some blends will inevitably land on the boundary between sky patches. This can be handled straightforwardly by defining overlapping patches, so that objects near the boundary are processed twice (once with each patch). One patch's processing is then selected to be canonical on an object-by-object basis. For successful deblending, this approach essentially relies on individual objects landing entirely within one patch, along with enough of any neighbors to deblend the primary object.

This requirement will not be met for some large galaxies (or pairs of large galaxies), and we'll likely have to use a different algorithm for these. The same may be true for some extremely large blends that do fit entirely within one patch, if necessary to keep deblender compute resources minimal. A multiscale approach seems natural here - start on a binned image of a larger sky area, and use this to generate templates for the largest objects. We then move to subimages at high resolution resolution to produce templates for smaller object, until we return to the regular pixel scale. The final linear fit for template coefficients ( $\alpha$ ) could then be done on a combination of regular pixels and binned superpixels, depending on which templates are active in a particular region, and may make use of sparse matrix methods. This approach may need to be iterative.

We may or may not want to use the same divisions for simultaneous fitting. By the time we reach the simultaneous fitting stage, we'll have some idea of the extents of children, and we may be able to find a divisions that require smaller overlap regions and/or a smaller number of objects with duplicate processing (by drawing boundaries that only touch a small number of compact objects). Patch boundaries will also be less important, as we'll be able to iterate directly over blend families (and hence only worry about sky patches for I/O). It may even be unnecessary to do any kind of divide-and-conquer for simultaneous fitting if we use sparse matrix methods and parallelize in a way that splits likelihood evaluation over multiple cores.

## 6 Models as Deblend Templates

Thus far we've considered simultaneous fitting as an optional stage following per-pixel deblending. We can also use the results of a simultaneous fit as the weighted templates  $(T\alpha)$  in a subsequent deblending step.

We've already highlighted model flexibility as an advantage of deblended measurement over simultaneous fitting, and this approach would remove some of that advantage, because the models used in simultaneous fitting are not as flexible as e.g. SDSS-style templates derived from symmetry arguments. It wouldn't remove the advantage entirely, because we apportion the original pixel values according to the relative template contributions rather than using the template values directly.

Even so, using simultaneous models as deblend templates does present some advantages over more flexible templates:

- We don't currently know how we're going to translate templates from coadds (where they're derived, at least for deep processing) to individual epochs, which involves both a change in PSF and coordinate system, and analytic models are one candidate.
- Using models provides a natural way to include prior information and constraints on the deblending, such as a requirement that deblending produce physically reasonable colors.
- It may be possible to propagate cross-object uncertainty estimates from a simultaneous fit into the deblend and hence moments-based measurements. A straightforward (but expensive) approach would be to repeat the suite of moments-based measurements on deblended pixels derived from model parameters at each sample point in a Monte Carlo simultaneous fit.

# 7 Variability, Transients, and Solar System Objects

When moving from coadd-based measurement to multi-epoch measurement, we need to consider how to deal with objects that are not static. We've already discussed moving point sources a bit, but we should clarify that this refers to slowly moving point sources – essentially, stars with proper motion and parallax. There are two distinguishing factors between these and faster solar system objects from an algorithmic perspective:

• They will be blended with the same neighbors in every epoch. As a result, we can model them simultaneously with galaxies using the same patch of sky in all epochs.

 We can detect faint moving stars below the single-epoch detection limit either by directly coadding all images or by coadding images with very small shifts.

Variable stars and quasars, which are present in every epoch with a different flux, can also be treated the same way; as discussed below, it is not clear at what stage we should model the variability, but it's reasonable to model them at every epoch in at least roughly the same part of the sky.

Transient events that affect only a small fraction of exposures but don't move are some what more difficult to handle. Many of these will be straightforwardly detected in single-epoch difference images, and others will be detected in special coadds that cover only limited epochs. However, we probably want to mask and reject these objects entirely when building coadds, so it will be impossible to deblend them there. Instead, we'll have to add them back in when we transition to multi-epoch measurement (which should be straightforward, as their positions will be known, and we'll assume they're point sources).

Fast moving solar system objects will similarly be detected in single-epoch difference images, and have orbits determined from these detections. We'll also want to mask and reject them from coadds. We still want to include them in multi-epoch measurement, both to ensure overlapping static objects are handled correctly and to measure flux as a function of time for the moving objects. We'll use a trailed model for at these (which of course approaches a point source as the speed decreases).

Extended variable or transient objects such as comets and supernova light echoes will be much harder to model or otherwise deblend in multi-epoch measurement, and our assumption for now is that these will be best analyzed via difference images, and hence in coaddition and multi-epoch measurement we'll simply mask them out.

# 8 LSST Pipeline Straw-Man Proposal

The above sections describe a number of algorithmic options that can be combined in myriad ways. In this section, we describe (at summary level) a full baseline pipeline and a few top-priority alternatives. The baseline plan is outlined in Figure 1, with details described in the next section and alternatives in Section 8.2.

#### 8.1 Baseline

The first major processing stage described here is the Deblender [P1], which we imagine as an algorithm very similar to that the SDSS *Photo* deblender described in Section 3, likely using a symmetry ansatz to define templates. The inputs will be a detection catalog containing merged Footprints and Peaks from all detection images [D1], and at least one coadd image per filter [D2]. We may have multiple input coadds for each filter, representing different depth vs. resolution tradeoffs or different epochs, and possibly some coadds that represent

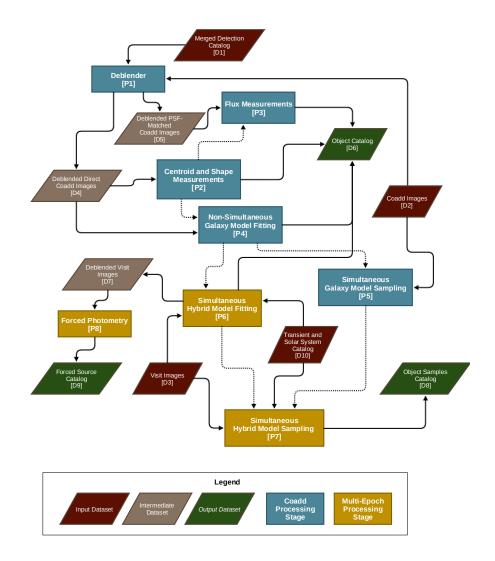


Figure 1: Baseline Pipeline for Blended Measurement

combinations of data from different filters. The details of these inputs and the parallelization and data flow within the deblender itself are beyond the scope of this document. The outputs are deblended pixel values for both direct [D4] and PSF- matched [D5] coadds (generated by sequentially replacing neighbors with noise, as described in Section 3).

These deblended coadds are used for three different groups of measurement algorithms, which we split here into separate processing stages mostly for clarity in their inputs and outputs (they may be run together):

- We start with centroid and moments-based shape measurements [P2] on deblended direct coadds [D4]. This includes all the standard centroiders as well as adaptive second moments. For each centroid or shape measurement, we'll define a single cross-filter output, either by selecting one filter (or combination of filters) as canonical or using algorithms that make use of all data from all filters.
- These consistent cross-filter centroid and shape measurements are used as inputs for traditional flux measurements [P3] on deblended PSF-matched coadds [D5]. These will include at least a sequence of aperture fluxes at predetermined radii as well as Kron and Petrosian fluxes.
- We'll also fit galaxy models [P4] to the deblended direct coadd images [D4] (fitting each object separately, of course, since these are deblended pixel values). Because the galaxy models become point sources at the zero radius limit, and there's no variability or astrometric information on the coadd, there'd be no point to additionally fitting a moving and/or variable point source model at this stage. We'll do at least one fit that uses the same structural (non-amplitude) parameters in all filters, to allow the model fluxes to be useful as for galaxy colors (see 8.2.3 for an alternative). We may also perform completely independent model fitting each each filter.

All three measurement stages will have some outputs that are included in the final object catalog [D6], but some may have temporary outputs that are used only to feed other measurement stages (indicated by the dotted lines in Figure 1). At this level of detail, we consider each of these measurements to have access to coadds from all filters, to allow forced measurements in one filter that depend on non-forced measurements in another. Whether this is done by giving algorithms access to all filters simultaneously or processing filters in serial (possibly more than once) is beyond the scope of this paper.

The last stage of measurement on coadds is simultaneous Monte Carlo sampling [P5], on the original (not deblended) direct coadds [D2]. We will again use galaxy models here, though they may not be the same as those used in [P4]. The outputs from this stage are used only as inputs to a multi-epoch sampling step [P7], and are essentially just a performance optimization.

In multi-epoch mode, we use hybrid models (as described in Section 4.1) and consider all objects in a blend simultaneously, first fitting with an optimizer [P6]

and then Monte Carlo sampling [P7]. In both cases we will fit to multiple filters simultenously (though perhaps not all filters), and only allow the flux to vary between filters (i.e. the models will not support variability – but see also 8.2.4 for an alternative). As in the coadd fitting, the structural parameters of the galaxy models will be required to be the same in each filter. As discussed in §5.2.2 of the DPDD, the most important use case for the Monte Carlo samples [D8] is shear estimation for gravitational lensing, but we anticipate it being useful for any study of faint objects for which unbiased population statistics are more important than precise measurements of individual objects. The optimizer-based fitting is should produce our best astrometric measurements for fainter stars, and may yield better galaxy photometry and morphology measurements than the deblended coadd fitting.

The simultaneous optimizer-based fit will also be used as templates for another round of deblending (as in Section 6), this time producing deblended pixel values for individual visits [D7]. These will be used for forced PSF photometry (DPDD §5.2.4) at the per-epoch positions determined from the simultaneous multi-epoch fit. This will populate the forced source catalog [D9], which represent our best estimates of the lightcurves of faint variable objects. We use positions from multi-epoch fitting to consistently handle stars with significant proper motions, and we perform only PSF photometry, as the vast majority of variable objects are indeed point sources.

For all multi-epoch measurements, we include models for transient and fast-moving objects detected in difference images [D10], as described in Section 7. These models will have a free flux parameter in each epoch, but will have centroids fixed at the position determined from detection image(s).

### 8.2 Possible Modifications

#### 8.2.1 Likelihood Coadds

Likelihood coadds (also known as Kaiser coadds or detection maps) present an intriguing but untested alternative to direct coadds. They represent an optimal combination of images in both image quality and S/N (impossible with direct coadds), but cannot be interpreted in the same way as traditional coadds and single-epoch images, requiring completely new algorithms for all operations performed on them. As a result, making use of likelihood coadds may require considerably more human effort, but it could reduce the need for multi-epoch processing (but probably not eliminate it). If they prove viable, likelihood coadds would replace direct coadds in most or all of the places the latter are currently used.

Evaluation of likelihood coadds will begin simply with analytical calculations and a small-scale prototype that operates only on postage stamp images. An evaluation of whether a full-scale optimized implemention is useful will be determined later.

### 8.2.2 Model Fluxes on PSF-Matched Coadds

Forward fitting of galaxy models only formally accounts for differences in PSF size across filters when the galaxy model is flexible enough to capture the true morphology of the galaxy being fit – a condition that is never fully met in practice. The best galaxy colors may thus require fitting to PSF-matched coadds instead of direct coadds, even though direct coadds may allow the model parameters to be constrained better.

If fitting to PSF-matched coadds produces better galaxy colors, we will do this in addition to fitting on direct coadds, as the latter will still produce better estimates of structural parameters and a better starting point for Monte Carlo sampling.

Given that PSF-matched coadds are needed for other purposes, and our galaxy fitting code must be robust enough that running on both direct coadds and PSF-matched coadds will require no new code, we will have the opportunity to evaluate both options in this area extensively before selecting one for final tuning. The development of fast metrics to evaluate the quality of galaxy color measurements will be critically important.

### 8.2.3 Consistent Cross-Filter Galaxy Structural Parameters

Galaxies do not have the same morphology in each filter, but those the differences between wavelengths are typically subtle enough that colors have historically been measured using the same structural parameters in each filter; if the PSF is also the same in each filter, this guarantees a consistent color even if the morphology is not correct in any filter, because it selects the same (incomplete) subset of the galaxy's light in each filter.

It may also be possible to use more flexible models in which the structural parameters can vary between filters to produce a better estimate of the total flux of the galaxy (and colors from the total fluxes are of course consistent as well). This requires additional degrees of freedom in the fit, and the additional flexibility increases the danger that measurements will select an inconsistent subset of the galaxies light across filters. If we can provide external constraints on how much the structural parameters can vary between filters (e.g. via Bayesian priors trained on space-based data), we may be able to allow for these extra degrees of freedom in a realistic way, which should produce better total flux and morphology estimates as well as consistent colors. These colors may also be higher S/N than those measured using the same model across filters on PSF-matched coadds; this depends on how the extra degrees of freedom from including more parameters compares to the loss of information in PSF-matched coadds.

Evaluating the options here is largely a matter of ensuring the galaxy-fitting code is sufficiently flexible that slightly different models and priors can be tested easily. Again, we will need good metrics for quantifying the quality of galaxy color measurements.

### 8.2.4 Variability in Multi-Epoch Modeling

Our baseline plan for multi-epoch modeling assumes objects have the same flux in every epoch. This is obviously incorrect for many point sources and even some galaxies (due to low-level AGN), and we may produce better results by including variability in these models. Such models could produce better measurements of light curves than simple forced photometry (perhaps making a separate forced photometry stage unnecessary). They could also improve star/galaxy classification of blended objects, and hence blended overall, by applying the ansatz that flux that varies between epochs should be attributed to point sources.

The main problem with introducing variability into the models is that it introduces many more degrees of freedom into the fit, vastly increasing the dimensionality of the problem. Given the many types of variable objects, and the complexity of the light curves of any of these, it is essentially impossible to devise analytic models that could predict the flux from just a few parameters; it will almost certainly be necessary to include an additional amplitude parameter for each epoch being fit. Because the model is linear in these parameters, however, their likelihood with all other parameters held fixed is exactly Gaussian, and this may enable us to marginalize analytically over these amplitudes while exploring the rest of the parameter space, while still retaining enough information to reconstruct the full joint distribution. This will require defining a Bayesian prior on the vector of amplitudes, though this could be based simply on the deviation from the mean flux rather than the distribution of fluxes as a function of time.

The simplest way to include variability in the models is to just add one amplitude parameters to the hybrid model when it is in moving-point source mode. This doesn't fully account for galaxies with AGN, however:

- In cases where the AGN flux dominates the total galaxy flux (i.e. quasars), this model would likely prefer a moving point-source model, ignoring extended flux from the galaxy even if it was detectable.
- In cases where the extended flux is comparable to or dominant over the AGN, these models would likely prefer the non-variable galaxy model, and treat the variability as noise. Because this "noise" would be inconsistent with the noise model we use to construct the likelihood, our any estimates of goodness-of-fit. Of course, this is also what happens (on a larger scale) when none of our models include variability.

A potential solution this problem would be to use a hybrid model that is a linear combination of a variable moving point source and static galaxy model, rather than hybrid model that transitions between the two. While this would have the same number of parameters overall, it would have more active at any time, truly increasing the dimensionality of the fit, though it would simplify the topology of that space significantly. More importantly, it would require the computationally expensive evaluation of a galaxy model for all likelihood evaluations, even when the evidence strongly suggests a point source. As a result, this approach is

probably not feasible unless we can devise a clever way to speed up or avoid some of those model evaluations.

While variable models may or may not be used in the mainline processing, it will be necessary to implement the capability to fit them regardless, and not just to evaluate them for use in the mainline processing – this sort of modeling is likely to be an important category of Level 3 processing, as any science involving strongly lensed quasars or AGN in galaxy clusters will require modeling complex blends of variable point sources and galaxies.

### 8.2.5 Forced Photometry on Difference Images

Another way to improve blended measurement of variable sources could be to run forced photometry on difference images instead of the original visit images. Because the extend light from galaxies is static, this should reduce the complex deblending problem to an exactly-solvable problem involving only point sources.

The only problem with this approach is the additional complexity in understanding the image data: the noise properties and effective PSF of a difference image are much more complex than that of a single epoch image, as we probably can't afford to simply ignore contributions from the template image.

This approach probably has the highest ceiling of any method for measuring variable blended sources, but it is untested and the mathematical formalism has yet to be developed.

### 8.2.6 Deblend Template Translation

Instead of using the models produced by simultaneous fitting as deblend templates for forced photometry, it may be possible to "translate" the deblend templates produced on the coadds to individual visit images. This translation would involve reconvolving to a different PSF (which will be a deconvolution for some images), and transforming to a new coordinate system. In fact, some sort of deblend translation code will have to exist even if we do not take this approach for forced photometry, in order to construct consistently-deblended direct and PSF-matched coadds (though that translation would involve only convolution to a larger PSF, assuming the deblending is done originally on direct coadds).

Transformation to a new coordinate system is just a matter of resampling, but reconvolution to a new PSF is trickier, at least when deconvolution may be involved. One possibility would be to use the same matching kernel algorithms used to build difference image; while these do not perform as well when matching a large PSF to a smaller one, they can deconvolve to a small degree.

We could also use regularized deconvolution techniques (e.g. sparse wavelet transforms) to construct deconvolved templates (still using symmetry arguments rather than analytic models), and then convolve them with the appropriate PSF for the image to be deblended, whether that's a coadd or a single-epoch image. A key point in this approach is that the deblended template need not match the true deconvolved source morphology (though clearly that is advantageous), or even be related to the pixel data in any statistically rigorous way; any template

that, when convolved with the PSF, approximates the as-observed morphology could work.

Translated deblending has the potential to better capture galaxy morphology than the simultaneous fitting approach we propose as the baseline, simply because the translated deblend templates will have more flexibility than the analytic models used in fitting. On the other hand, translated deblend templates will be limited by the quality of the coadd and their inability to account for proper motions.